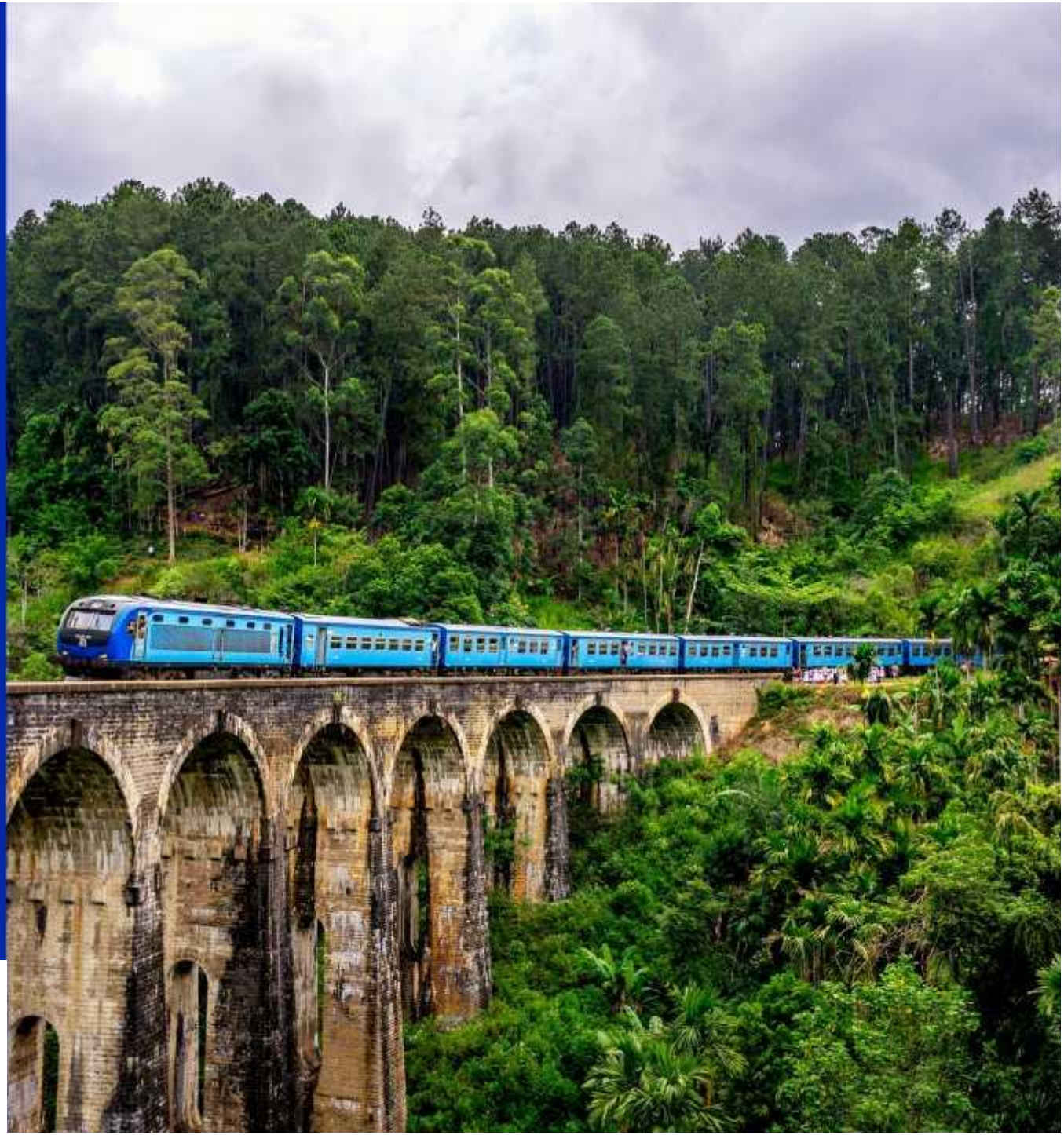
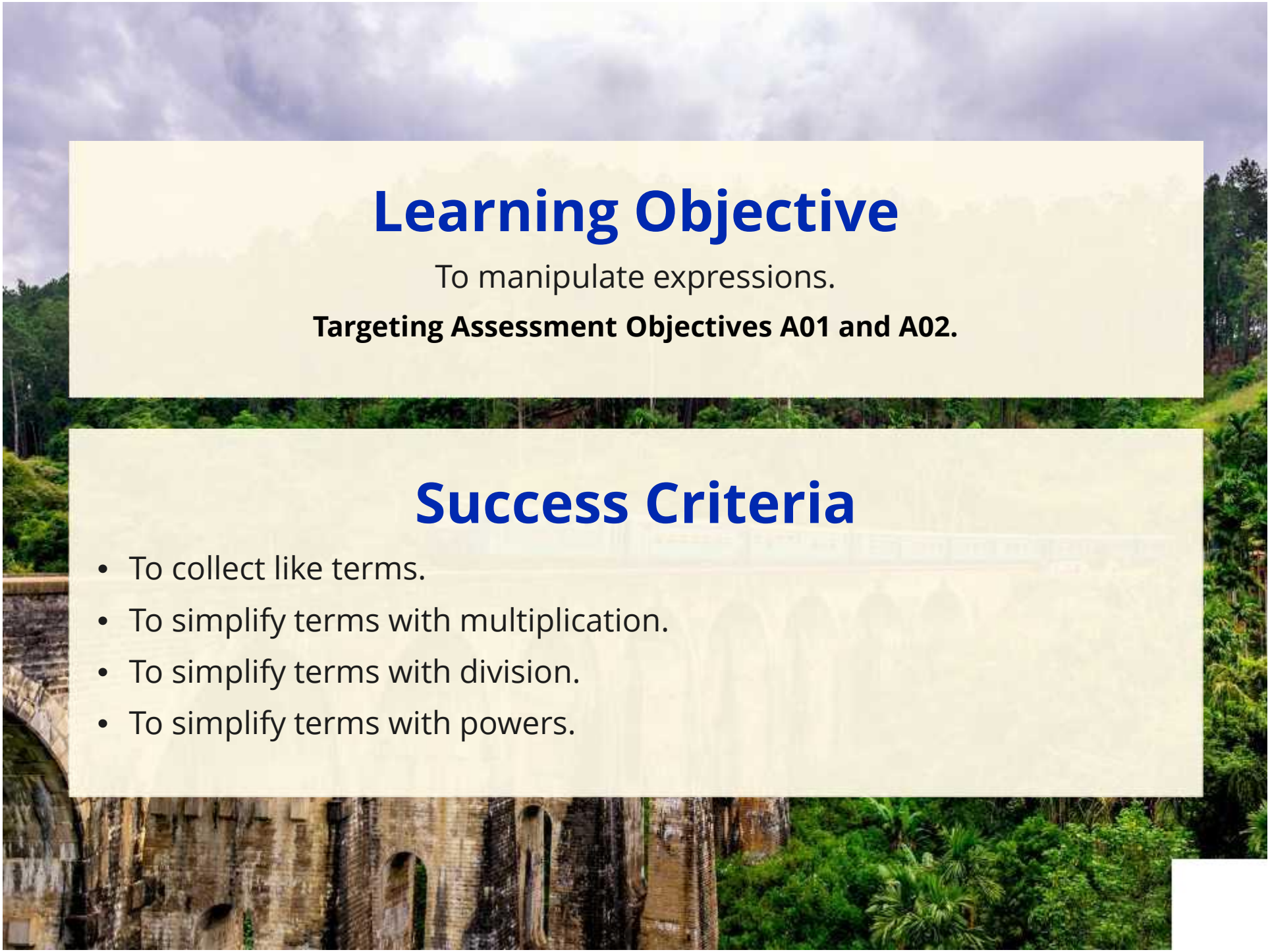
Simplifying Algebraic Expressions

Questions - Set 2





Learning Objective

To manipulate expressions.

Targeting Assessment Objectives A01 and A02.

Success Criteria

- To collect like terms.
- To simplify terms with multiplication.
- To simplify terms with division.
- To simplify terms with powers.

Your Choice

Collecting Terms

Simplifying with Multiplication

Simplifying with Division

Simplifying with Powers

Simplifying Mixed Questions

Collecting Terms

Example 1: Write the following expression in its simplest terms:

$3x + 5y - 2y + 7x$

Start by rearranging the expression to collect the like terms (xs and ys). Remember, the add and subtract signs should be collected with the terms **after** them, not before.

$3x + 7x + 5y - 2y$

Next, simplify each term:

$3x + 7x = 10x$

$5y - 2y = 3y$

$3x + 7x + 5y - 2y = \mathbf{10x + 3y}$

Collecting Terms

Example 2: Write the following expression in its simplest terms:

$$2x^2 - 3x + 5x + x^2$$

As before, start by collecting the like terms. Remember, to collect terms, they must be identical. x^2 and x are not like terms.

$$2x^2 + x^2 - 3x + 5x$$

Then, simplify:

$$2x^2 + x^2 - 3x + 5x = \mathbf{3x^2 + 2x}$$

Collecting Terms: Your Turn

Simplify where possible:

1. $2a + 3b + a + 5b =$
2. $3 + 2x + 4x + 4 =$
3. $2z + 4y + z + 3y =$
4. $6c + 4d - 2c + 3d =$
5. $2m + 6 - m - 3 =$
6. $5p - 2q - 4p + q =$
7. $2x + 3x^2 + 3x - x^2 =$
8. $2ab + 2a + 5ab + 2a =$
9. $3a + 2a^2 - 3a^3 =$
10. $-3x - 5y - 2y - 5x =$

Answers

Collecting Terms: Answers

Simplify where possible:

1. $2a + 3b + a + 5b = \mathbf{3a + 8b}$
2. $3 + 2x + 4x + 4 = \mathbf{6x + 7}$
3. $2z + 4y + z + 3y = \mathbf{7y + 3z}$
4. $6c + 4d - 2c + 3d = \mathbf{4c + 7d}$
5. $2m + 6 - m - 3 = \mathbf{m + 3}$
6. $5p - 2q - 4p + q = \mathbf{p - q}$
7. $2x + 3x^2 + 3x - x^2 = \mathbf{2x^2 + 5x}$
8. $2ab + 2a + 5ab + 2a = \mathbf{7ab + 4a}$
9. $3a + 2a^2 - 3a^3 = \mathbf{3a + 2a^2 - 3a^3}$ (cannot be simplified)
10. $-3x - 5y - 2y - 5x = \mathbf{-8x - 7y}$

Home

Simplifying with Multiplication

Example 1: Simplify the following expression:

$2 \times 4a$

To find 2 lots of $4a$, we multiply the 4 by 2:

$2 \times 4a = \mathbf{8a}$

Example 2: Simplify the following expression:

$3a \times 5b$

In this case, we have two different algebraic terms. Because they're different, we multiply the numbers but leave the algebraic terms:

$3a \times 5b = \mathbf{15ab}$

Simplifying with Multiplication

Example 3: Simplify the following expression:

$a^2 \times a^3$

When multiplying the same algebraic term, we add the indices:

$a^2 \times a^3 = a^{2+3} = \mathbf{a^5}$

Example 4: Simplify the following expression:

$2a^3b^2 \times 3a^2b$

In longer questions like this, look at the numbers and each algebraic term separately:

$2 \times 3 = 6$

$a^3 \times a^2 = a^{3+2} = a^5$

$b^2 \times b = b^{2+1} = b^3$ (remember, $b = b^1$)

$2a^3b^2 \times 3a^2b = \mathbf{6a^5b^3}$

Simplifying with Multiplication

Example 5: Simplify the following expression:

$$4a^3b^5c^{-2}d^7 \times 5a^3bc^{-3}d^{-2}$$

It doesn't matter how long the question is. You can still do it by breaking it down into numbers and algebraic terms:

$4 \times 5 = 20$

$a^3 \times a^3 = a^6$

$b^5 \times b = b^6$

$c^{-2} \times c^{-3} = c^{-5}$

$d^7 \times d^{-2} = d^5$

$$4a^3b^5c^{-2}d^7 \times 5a^3bc^{-3}d^{-2} = \mathbf{20a^6b^6c^{-5}d^5}$$

Simplifying with Multiplication: Your Turn

Simplify:

1. $2ab \times 3c =$
2. $5x \times 8y =$
3. $5mn \times 6pq =$
4. $2a^2 \times 4a =$
5. $10mp^2 \times 3m^3p^2 =$
6. $2x^2 \times x^5 =$
7. $2b^3 \times 0.5a^2 =$
8. $6m^5p \times 4m^4p^2 =$
9. $2mn \times 7mn =$
10. $7ac \times -2ca =$

Answers

Simplifying with Multiplication: Answers

Simplify:

1. $2ab \times 3c = \mathbf{6abc}$
2. $5x \times 8y = \mathbf{40xy}$
3. $5mn \times 6pq = \mathbf{30mnpq}$
4. $2a^2 \times 4a = \mathbf{8a^3}$
5. $10mp^2 \times 3m^3p^2 = \mathbf{30m^4p^4}$
6. $2x^2 \times x^5 = \mathbf{2x^7}$
7. $2b^3 \times 0.5a^2 = \mathbf{a^2b^3}$
8. $6m^5p \times 4m^4p^2 = \mathbf{24m^9p^3}$
9. $2mn \times 7mn = \mathbf{14m^2n^2}$
10. $7ac \times -2ca = \mathbf{-14a^2c^2}$

Home

Simplifying with Division

Example 1: Simplify the following expression:

$$14x \div 2$$

As the only thing both terms have in common are numbers, we just divide 14 by 2, leaving the x alone:

$$14x \div 2 = \mathbf{7x}$$

Example 2: Simplify the following expression:

$$\frac{b}{b}$$

First, remember that the line in the middle of the fraction means divide. This question is the same as asking $b^7 \div b^4$.

When dividing algebraic terms, we subtract the indices:

$$b^7 \div b^4 = b^{7-4} = \mathbf{b^3}$$

Simplifying with Division

Example 3: Simplify the following expression:

$$\frac{18a\ b}{3ab}$$

In questions like this, treat the numbers and each algebraic term separately:

$18 \div 3 = 6$

$a^5 \div a = a^{5-1} = a^4$ (remember, $a = a^1$)

$b^2 \div b^5 = b^{2-5} = b^{-3}$

$$\frac{18a\ b}{3ab} = \mathbf{6a^4b^{-3}}$$

Simplifying with Division

Example 4: Simplify the following expression:

$$\frac{12n}{6n^{-1}}$$

Watch out for negative signs when dividing by negative indices:

$12 \div 6 = 2$

$n^5 \div n^{-1} = n^{5-(-1)} = n^6$

$$\frac{12n}{6n^{-1}} = \mathbf{2n^6}$$

Simplifying with Division

Example 5: Simplify the following expression:

$$\frac{18a\ b}{24a\ b}$$

Sometimes, when simplifying a division, you will still be left with a fraction. You can tell that is going to be the case here because 24 is not a factor of 18, and the power of b on the denominator is higher than the power of b on the numerator.

In this case, we'll consider the whole fraction:

$$\frac{18a\ b}{24a\ b} = \frac{\mathbf{3a}}{\mathbf{4b^2}}$$

$\frac{18}{24}$ simplifies to $\frac{3}{4}$

$\frac{b}{b}$ cancels to b^2 on the denominator

$\frac{a}{a}$ cancels to a on the numerator

Simplifying with Division: Your Turn

Simplify:

1. $\frac{x}{x} =$
2. $a^5 \div a^3 =$
3. $\frac{12mp^4}{2p} =$
4. $\frac{16x\ y}{4x\ y} =$
5. $8rst \div 4st =$
6. $2m^2p \div 10mp =$
7. $\frac{4x}{2x} =$
8. $\frac{5a\ b}{abc} =$
9. $\frac{15m\ n}{5mn} =$
10. $\frac{12ab^2c}{6a\ c} =$

Answers

Simplifying with Division: Answers

Simplify:

1. $\frac{x}{x} = \boldsymbol{x}$
2. $a^5 \div a^3 = \boldsymbol{a^2}$
3. $\frac{12mp^4}{2p} = \boldsymbol{6mp^2}$
4. $\frac{16x\ y}{4x\ y} = \boldsymbol{4xy}$
5. $8rst \div 4st = \boldsymbol{2r}$
6. $2m^2p \div 10mp = \boldsymbol{\frac{m}{5}}$
7. $\frac{4x}{2x} = \boldsymbol{2x}$
8. $\frac{5a\ b}{abc} = \boldsymbol{\frac{5a}{c}}$
9. $\frac{15m\ n}{5mn} = \boldsymbol{3mn^3}$
10. $\frac{12ab^2c}{6a\ c} = \boldsymbol{\frac{2b}{a}}$ **or** $\boldsymbol{2a^{-2}b^2c^{-3}}$

Home

Simplifying with Powers

Example 1: Simplify the following expression:

$(x^5)^2$

When raising a power to a power, you multiply the indices:

$(x^5)^2 = x^{5 \times 2} = \boldsymbol{x^{10}}$

Example 2: Simplify the following expression:

$(3x^4)^3$

In this question, you have to raise both the algebraic term and the number to the power of 3. Do this separately:

$(x^4)^3 = x^{4 \times 3} = x^{12}$

$3^3 = 27$

$(3x^4)^3 = \boldsymbol{27x^{12}}$

Simplifying with Powers: Your Turn

Simplify:

1. $(x^4)^2 =$
2. $(3x^3)^2 =$
3. $(x^4)^3 =$
4. $(4^x)^3 =$
5. $(5m^2)^2 =$
6. $(3yz^2)^2 =$
7. $(bc^3)^4 =$
8. $(4a^2)^3 =$
9. $(2m^4)^5 =$
10. $(\frac{1}{3}x^2)^2 =$

Answers

Simplifying with Powers: Answers

Simplify:

1. $(x^4)^2 = \mathbf{x^8}$
2. $(3x^3)^2 = \mathbf{9x^6}$
3. $(x^4)^3 = \mathbf{x^{12}}$
4. $(4^x)^3 = \mathbf{4^{3x}}$
5. $(5m^2)^2 = \mathbf{25m^4}$
6. $(3yz^2)^2 = \mathbf{9y^2z^4}$
7. $(bc^3)^4 = \mathbf{b^4c^{12}}$
8. $(4a^2)^3 = \mathbf{64a^6}$
9. $(2m^4)^5 = \mathbf{32m^{20}}$
10. $(\frac{1}{3}x^2)^2 = \mathbf{\frac{1}{9}x^4}$

Home

Simplifying Mixed Questions: Your Turn

Simplify:

1. $z + 3y + 5z + 2y =$
2. $4x \times 8yz =$
3. $a^6 \div a^2 =$
4. $(5x^4)^2 =$
5. $3a + 5b - a + 4b =$
6. $4mp^3 \times 5m^2p^5 =$
7. $\frac{12x\ y}{4x\ y} =$
8. $(2m^3)^6 =$
9. $4x + 2x^2 + x - 3x^2 =$
10. $4b^4 \times -0.5a^3b =$

Answers

Simplifying Mixed Questions: Answers

Simplify:

1. $z + 3y + 5z + 2y = \mathbf{6z + 5y}$
2. $4x \times 8yz = \mathbf{32xyz}$
3. $a^6 \div a^2 = \mathbf{a^4}$
4. $(5x^4)^2 = \mathbf{25x^8}$
5. $3a + 5b - a + 4b = \mathbf{2a + 9b}$
6. $4mp^3 \times 5m^2p^5 = \mathbf{20m^3p^8}$
7. $\frac{12x\ y}{4x\ y} = \mathbf{3x^3}$
8. $(2m^3)^6 = \mathbf{64m^{18}}$
9. $4x + 2x^2 + x - 3x^2 = \mathbf{5x - x^2}$
10. $4b^4 \times -0.5a^3b = \mathbf{-2a^3b^5}$

Home